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SEPARATION AXIOMS BY $\psi^*\alpha$ -CLOSED SETS

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ABSTRACT

In this paper separation axioms of $\psi^*\alpha$ -closed sets namely ${}_{\psi^*\alpha}T_c$-space, ${}_{\psi^*\alpha}T_\alpha$-space, ${}_{g\alpha}T_{\psi^*\alpha}$ -space, ${}_{\alpha g}T_{\psi^*\alpha}$ -space and ${}_{\psi g}T_{\psi^*\alpha}$-space are introduced and their properties are analyzed.

KEYWORDS: αg -closed sets, gα- closed sets, ψg-closed sets and $\psi^*\alpha$ -closed sets

INTRODUCTION

Njastad [7] introduced the concept of an α-open sets. Levine [5] introduced the notion of g-closed sets in topological spaces and studied their basic properties. Ramya and Parvathi [8] introduced a new concept of generalized closed sets called $\psi\hat{g}$ -closed sets and ψg-closed sets in topological spaces. A new class of generalized closed sets called $\psi^*\alpha$ -closed sets in topological spaces using ψg-closed sets was introduced in 2016 by Balamani and Parvathi[1]. The objective of this paper is to contribute the separation axioms by $\psi^*\alpha$ -closed sets and study their properties. Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are specified, unless otherwise mentioned.

PRELIMINARIES

Definition 2.1 A subset A of a topological space (X, τ) is called

1) generalized closed set (briefly g-closed) [5] if cl(A) $\subseteq$ U whenever A$\subseteq$U and U is open in (X, τ).
2) semi-generalized closed set (briefly sg-closed)[2] if scl(A) $\subseteq$ U whenever A$\subseteq$U and U is semi- open in (X, τ).
3) ψ-closed set [10] if scl(A) $\subseteq$ U whenever A$\subseteq$U and U is sg-open in (X, τ).
4) ψg-closed set [8] if ψcl(A) $\subseteq$ U whenever A$\subseteq$U and U is open in (X, τ).
5) $\psi^*\alpha$ -closed set[1] if αcl(A) $\subseteq$ U whenever A$\subseteq$U and U is ψg -open in (X, τ).
6) The closure operator of $\psi^*\alpha$-closed set is defined as $\psi^*\alpha cl(A) = \cap \{F \subseteq X : A \subseteq F$ and F is $\psi^*\alpha$ -closed in (X,τ)}[1]

Definition 2.2 A topological space (X, τ) is said to be a

(i) T_b- space if every gs-closed subset of (X, τ) is closed in (X, τ). [4]
(ii) T_d- space if every gs-closed subset of (X, τ) is g-closed in (X, τ). [4]
(iii) T_c- space if every gs-closed subset of (X, τ) is g^*-closed in (X, τ). [9]
(iv) ${}_{\alpha}T_b$- space if every αg-closed subset of (X, τ) is closed in (X, τ). [3]
(v) ${}_{\alpha}T_{1/2}$ -space if every gα-closed subset of (X, τ) is α-closed in (X, τ). [6]
(vi) ${}_{1/2}T_\alpha$-space if every αg-closed subset of (X, τ) is α-closed in (X, τ). [6]
(vii) ${}_{\alpha}T_d$- space if every αg-closed subset of (X, τ) is g-closed in (X, τ). [3]
(viii)) ${}_{\alpha}T_c$- space if every αg-closed subset of (X, τ) is g^*-closed in (X, τ). [9]
(ix) ${}^{*}T_{1/2}$ – space if every g-closed subset of (X, τ) is g^*-closed in (X, τ). [9]
(x) α-space if every α-closed subset of (X, τ) is closed in (X, τ). [7]
(xi) ψ-space if every ψ-closed subset of (X, τ) is closed in (X, τ).[10]

SEPARATION AXIOMS

Definition 3.1 A topological space (X, τ) is said to be a
(i) $_{\psi*\alpha}T_c$-space if every $\psi^*\alpha$ closed subset of (X, τ) is closed in (X, τ).
(ii) $_{\psi*\alpha}T_\alpha$-space if every $\psi^*\alpha$ closed subset of (X, τ) is α- closed in (X, τ).
(iii) $_{g\alpha}T_{\psi*\alpha}$ - space if every gα- closed subset of (X, τ) is $\psi^*\alpha$-closed in (X, τ).
(iv) $_{\alpha g}T_{\psi*\alpha}$ -space if every αg- closed subset of (X, τ) is $\psi^*\alpha$-closed in (X, τ).
(vii) $_{\psi g}T_{\psi*\alpha}$-space if every ψg- closed subset of (X, τ) is $\psi^*\alpha$-closed in (X, τ).

Proposition 3.2 Every $_{\psi*\alpha}T_c$-space is a $_{\psi*\alpha}T_\alpha$-space but not conversely.

Proof: Let A be a $\psi^*\alpha$-closed set in (X, τ). Since (X, τ) is a $_{\psi*\alpha}T_c$-space, A is closed in (X, τ). Since every closed set is α-closed, A is α-closed in (X, τ). Hence (X, τ) is $_{\psi*\alpha}T_\alpha$-space.

Example 3.3 Let X={a, b, c} with topology τ={ϕ,{a},{X}}.Then (X, τ) is a $_{\psi*\alpha}T_\alpha$-space but not $_{\psi*\alpha}T_c$-space, since the subset {b} is $\psi^*\alpha$ -closed but not closed in (X, τ).

Theorem 3.4 If (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space and an α - space, then it is a $_{\psi*\alpha}T_c$ – space.

Proof: Let A be a $\psi^*\alpha$ -closed set in (X, τ).Since (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space, A is α - closed in (X, τ) Since (X, τ) is an α - space, A is closed in (X, τ) Hence (X, τ) is a $_{\psi*\alpha}T_c$ - space.

Theorem 3.5 If (X, τ) is a $_{\psi*\alpha}T_c$ - space (resp. $_{\psi*\alpha}T_\alpha$ - space) then $\psi^*\alpha$cl(B) = cl(B)(resp. αcl(B)) for each subset B of (X, τ).

Proof: Since (X, τ) is a $_{\psi*\alpha}T_c$ - space (resp. $_{\psi*\alpha}T_\alpha$ - space). Since every closed (resp. α - closed) set is $\psi^*\alpha$ - closed in (X, τ), $\psi^*\alpha$C((X, τ) = C(X, τ) (resp.αC(X, τ)). Hence $\psi^*\alpha$cl(B) = cl(B) (resp. αcl(B)) for each subset B of (X, τ).

Theorem 3.6 If (X, τ) is a $_{\psi*\alpha}T_c$-space, then for each x∈X either {x} is ψg - closed or open.

Proof: Let x∈X and suppose {x} is not ψg - closed in (X, τ). Then X-{x} is not ψg - open. Hence X is the only ψg - open set containing X - {x}. This implies that X -{x} is a $\psi^*\alpha$- closed set in (X, τ). Since (X, τ) is a $_{\psi*\alpha}T_c$-space, X-{x} is a closed in (X, τ) or equivalently {x} is open in (X, τ).

Theorem 3.7 For a space (X, τ) the following conditions are equivalent
(i) (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space
(ii) For each x∈X, {x} is either α - open or ψg - closed.

Proof: (i) ⇒ (ii) Let x∈X and suppose {x} is not ψg - closed in (X, τ). Then X-{x} is not ψg-open. Hence X is the only ψg - open set containing X-{x}. So X -{x} is a $\psi^*\alpha$- closed set in (X, τ). Since (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space, X-{x} is an α-closed set in (X, τ) or equivalently {x} is an α - open set in (X, τ).

(ii) ⇒ (i) Let A be a $\psi^*\alpha$ - closed set in (X, τ) and x∈ αcl(A). We show that x∈A for the following two cases.

Case 1: Assume that {x} is α - open. Then X-{x} is α - closed. If x∉A, then A ⊆ X-{x}. Since x∈αcl(A), we have x ∈X-{x}, which is a contradiction. Hence x∈A.

Case 2: Assume that {x} is ψg - closed and x∉A. Then αcl(A)-A contains a ψg - closed set {x}.This contradicts Theorem 4.5[1]. Therefore x ∈A.

Theorem 3.8 If (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space, then for every subset A of (X, τ) $\psi^*\alpha$cl(A) is α - closed in (X, τ).

Proof: By definition, $\psi^*\alpha$cl(A)= ∩ {F⊆X: A ⊆F and F is $\psi^*\alpha$ -closed in (X,τ)}. Since (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space, $\psi^*\alpha$cl(A) is α - closed in (X, τ). Since every α - closed set is $\psi^*\alpha$ - closed set in (X, τ), $\psi^*\alpha$cl(A) is α - closed in (X, τ).

Proposition 3.9 Every $_\alpha T_{1/2}$ -space (resp. $_{1/2}T_\alpha$-space) is a $_{\psi*\alpha}T_\alpha$-space but not conversely.

Proof: Let (X, τ) be an $T_{1/2}$ -space(resp. $_{1/2}T_\alpha$-space) and let A be a $\psi^*\alpha$-closed set in (X, τ). By proposition 3.10[1] A is gα-closed (resp. αg-closed) in (X, τ). Since (X, τ) is an $_\alpha T_{1/2}$ -space(resp. $_{1/2}T_\alpha$-space), A is α-closed in (X, τ). Hence (X, τ) is $_{\psi*\alpha}T_\alpha$-space.

Example 3.10 Let X={a, b, c} with topology τ={ϕ,{a},{b, c},X}.Then (X, τ) is a $_{\psi*\alpha}T_\alpha$ - space but not an $_\alpha T_{1/2}$ -space and not a $_{1/2}T_\alpha$-space, since the subsets {b},{c},{a, b}and {a, c} are gα - closed and αg-closed but not α -closed in (X, τ)

Proposition 3.11 Every T_b-space is a $_{\psi*\alpha}T_\alpha$-space but not conversely.

Proof: Let (X, τ) be a T_b-space and let A be a $\psi^*\alpha$-closed set in (X, τ). By proposition 3.10[1] A is gs-closed in (X, τ). Since (X, τ) is a T_b-space, A is closed in (X, τ) and so it is α-closed in (X, τ). Hence (X, τ) is $_{\psi*\alpha}T_\alpha$-space.

Example 3.12 Let X={a, b, c} with topology τ={ϕ,{a},{b, c},X}.Then (X, τ) is a $_{\psi*\alpha}T_{\alpha}$ - space but not a T_b -space, since the subsets {b},{c},{a, b}and {a, c} are gs- closed but not closed in (X, τ).

Proposition 3.13 Every $_{\alpha}T_b$ -space is a $_{\psi*\alpha}T_{\alpha}$-space but not conversely.

Proof: Let (X, τ) be a $_{\alpha}T_b$ -space and let A be a $\psi^*\alpha$-closed set in (X, τ). By proposition 3.10[1] A is g-closed in (X, τ). Since (X, τ) is a $_{\alpha}T_b$ -space, A is closed in (X, τ) and so it is α-closed in (X, τ). Hence (X, τ) is $_{\psi*\alpha}T_{\alpha}$-space.

Example 3.14 Let X={a, b, c} with topology τ={ϕ,{a}, X}.Then (X, τ) is a $_{\psi*\alpha}T_{\alpha}$ - space but not a $_{\alpha}T_b$ -space, since the subsets {a, b}and {a, c} are αg- closed but not closed in (X, τ).

Proposition 3.15 Every ψ-space is a $_{\psi*\alpha}T_{\alpha}$-space but not conversely.

Proof: Let (X, τ) be a ψ-space and let A be a $\psi^*\alpha$-closed set in (X, τ). By proposition 3.24[1] A is ψ-closed in (X, τ). Since (X, τ) is a ψ-space, A is closed in (X, τ) and so it is α-closed in (X, τ). Hence (X, τ) is $_{\psi*\alpha}T_{\alpha}$-space.

Example 3.16 Let X={a, b, c} with topology τ={ϕ,{a}, X}.Then (X, τ) is a $_{\psi*\alpha}T_{\alpha}$ - space but not a ψ -space, since the subsets {b} and {c} are ψ- closed but not closed in (X, τ).

Remark 3.17 The following examples show that $_{\psi*\alpha}T_{\alpha}$-space is independent from $_{g\alpha}T_{\psi*\alpha}$ –space, $_{\alpha g}T_{\psi*\alpha}$ –space and $_{\psi g}T_{\psi*\alpha}$ –space

Example 3.18 Let X = {a, b, c} with topology τ={ϕ,{a},{b, c},X}.Then (X, τ) is a $_{\psi*\alpha}T_{\alpha}$-space but not a $_{g\alpha}T_{\psi*\alpha}$ –space, not a $_{\alpha g}T_{\psi*\alpha}$ –space and not a $_{\psi g}T_{\psi*\alpha}$ -space, since the subsets {b},{c}, {a, b}and {a, c} are gα-closed, αg -closed and ψg-closed but not $\psi^*\alpha$ -closed in (X, τ).

Example 3.19 Let X={a, b, c} with topology τ={ϕ, {a, b},X}.Then (X, τ) is a $_{g\alpha}T_{\psi*\alpha}$ -space, $_{\alpha g}T_{\psi*\alpha}$ –space and $_{\psi g}T_{\psi*\alpha}$ -space but not a $_{\psi*\alpha}T_{\alpha}$-space, since the subsets {a, c } and {b, c} are $\psi^*\alpha$ -closed but not α - closed in (X, τ).

Remark 3.20 The space $_{\psi*\alpha}T_{\alpha}$ is independent with α-space, $_{\alpha}T_d$ – space and $_{\alpha}T_c$-space as seen from the following example.

Example 3.21 Let X = {a, b, c} with topology τ={ϕ, {a, b},X}.Then (X, τ) is an α -space, $_{\alpha}T_d$ - space and $_{\alpha}T_c$-space but not $_{\psi*\alpha}T_{\alpha}$ - space, since the subsets {a, c} and {b, c} are $\psi^*\alpha$ - closed but not α - closed in (X, τ).

Example 3.22 Let X = {a, b, c} with topology τ={ϕ,{a},{a, b},X}.Then (X, τ) is a $_{\psi*\alpha}T_{\alpha}$ - space but not an α -space, $_{\alpha}T_d$ - space and $_{\alpha}T_c$-space, since the subset {b} is α - closed and αg-closed but not closed, g-closed and g^*-closed in (X, τ).

Remark 3.23 The following examples show that $_{\psi*\alpha}T_{\alpha}$ - space and $^*T_{1/2}$ - space are independent.

Example 3.24 Let X = {a, b, c} with topology τ={ϕ,{a, b},X}. Then (X, τ) is a $^*T_{1/2}$ - space but not a $_{\psi*\alpha}T_{\alpha}$ -space, since the subsets {a, c} and {b, c} are $\psi^*\alpha$ - closed but not α - closed in (X, τ).

Example 3.25 Let X = {a, b, c} with topology τ= {ϕ,{a},X}. Then (X, τ) is a $_{\psi*\alpha}T_{\alpha}$ - space but not a $^*T_{1/2}$ -space, since the subsets {b}, {c}, {a, b} and {a, c} are g - closed but not g^*- closed in (X, τ).

Proposition 3.26 Every $_{\alpha g}T_{\psi*\alpha}$ -space is a $_{g\alpha}T_{\psi*\alpha}$ –space but not conversely.

Proof: The proof follows from the fact that every gα -closed set is αg- closed.

Example 3.27 Let X={a, b, c} with topology τ={ϕ,{a},{a, b},X}.Then (X, τ) is a $_{g\alpha}T_{\psi*\alpha}$ -space but not $_{\alpha g}T_{\psi*\alpha}$ -space, since the subset {a, c} is αg- closed but not $\psi^*\alpha$ -closed in (X, τ).

Proposition 3.28 Every $_{\psi g}T_{\psi*\alpha}$ -space is a $_{g\alpha}T_{\psi*\alpha}$ –space and a $_{\alpha g}T_{\psi*\alpha}$ –space but not conversely.

Proof: The proof follows from the fact that every gα- closed set and αg-closed set is ψg- closed.

Example 3.29 Let X={a, b, c} with topology τ={ϕ,{a},{b},{a, b},X}.Then (X, τ) is a $_{g\alpha}T_{\psi*\alpha}$ -space and a $_{\alpha g}T_{\psi*\alpha}$ –space but not a $_{\psi g}T_{\psi*\alpha}$ –space, since the subsets {a} and {b} are ψg - closed but not $\psi^*\alpha$ -closed in (X, τ).

Remark 3.30 The spaces $_{g\alpha}T_{\psi*\alpha}$ -space, $_{\alpha g}T_{\psi*\alpha}$ –space and $_{\psi g}T_{\psi*\alpha}$ -space are independent with ψ - space.

Example 3.31 Let X = {a, b, c} with topology τ={ϕ,{a}, {b, c},X}.Then (X, τ) is a ψ -space but not a $_{g\alpha}T_{\psi*\alpha}$ –space, not a $_{\alpha g}T_{\psi*\alpha}$ –space and not a $_{\psi g}T_{\psi*\alpha}$ –space, since the subsets {b},{c},{a, b} and {a, c} are gα- closed, αg-closed and ψg- closed but not $\psi^*\alpha$ - closed in (X, τ).

Example 3.32 Let X = {a, b, c} with topology τ={ϕ,{a, b},X}.Then (X, τ) is a $_{g\alpha}T_{\psi*\alpha}$ –space, a $_{\alpha g}T_{\psi*\alpha}$ –space and a $_{\psi g}T_{\psi*\alpha}$ -space but not ψ -space, since the subsets {a, c} and {b, c} are ψ -closed but not closed in (X, τ).

REFERENCES

[1] Balamani,N and Parvathi,A., "Between α-closed sets and $\tilde{g}_{\alpha}$ -closed sets", International Journal of Mathematical Archive, Vol 7(6), pp.1-10, 2016.

[2] Bhattacharyya,P. and Lahiri, B.K., "Semi-generalized closed sets in topology", Indian J. Math., Vol.29, pp.376-382,1987.

[3] Devi, R. Balachandran, K and Maki, H., "generalized α-closed maps and α-generalized closed maps", Indian Journal of Pure and Applied Mathematics.,Vol.29, pp.37-49,1988.

[4] Devi, R. Maki, H. and Balachandran, K. Semi generalized closed maps and generalized semi closed maps", Mem. Fac. Sci. Kochi Univ. Ser. A. Math., Vol.14, pp.41 – 54, 1993.

[5]]Levine,N., "Generalized closed sets in topology",Rend. Circ. Math. Palermo,Vol. 19, pp.89 – 96,1970.

[6] Maki, H., Devi, R. and Balachandran, K., "generalized α - closed sets in topology", Bull.Fukuoka. Univ. Ed, part III, 42, pp.13-21,1993.

[7] Njastad, O., " On some classes of nearly open sets", Pacific J. Math., Vol.15, pp.961 – 970, 1965.

[8] Ramya,N.,and Parvathi,A., "$\psi\hat{g}$-closed sets in topological spaces", International Journal of Mathematical Archive- Vol.2(10), pp.1992 – 1996,2011.

[9] Veerakumar, M.K.R.S., "Between closed sets and g - closed sets", Mem. Fac. Sci. Koch.Univ. Ser. A, Math., Vol.21, pp.1-19,2000.

[10] Veera kumar, M.K.R.S, "Between semi-closed sets and semi-pre closed sets, Rend. Istit.Mat. Univ. Trieste, (ITALY) Vol.XXXXII, pp.25-41, 2000.